Investigation of Thin-Sheet Approaches to Simulate Beam Tube Losses.*

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Introduction

The beam tube in acceleration facilities is often made of very thin metal. During the switch-on or switch-off process of the dipole magnets, the fast changes in the magnetic field lead to eddy currents in the conductive beam tube. Consequently, electric losses emerge and the magnetic field inside the tube is perturbed. Both effects can be of importance. The purpose of this study is to accurately simulate the eddy current losses and their influence on the magnetic field by using the finite element method (FEM). Thin-sheet approaches are applied to avoid the high computational costs when the FEM is used with very small sheets like the beam tube. In a first step, different approaches from the literature are compared to a new method developed by the authors for a small test case. The focus of the comparison is put on the condition number of the final system matrix.

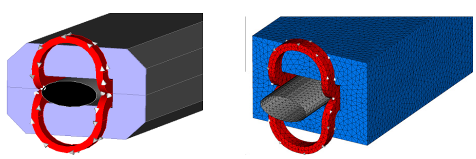


Figure 1: Model and reduced mesh of dipole magnet.

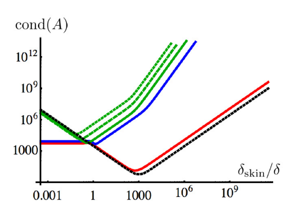
Thin-Sheet Approaches

Two main difficulties arise from very thin objects in the calculation domain. Automatic meshing often fails or ends up in inappropriate meshes. But even with a mesh found, the resulting system matrix is ill-conditioned and, consequently, hard to solve with iterative methods. The common idea of thin-sheet approaches is to replace the sheet volume by an interface and applying impedance transmission conditions (ITCs) between the two sides of the interface in order to reproduce the physical properties of the original sheet volume.

Many ITCs are given in the literature. The easiest way is to replace the interface by a perfect electric conductor which is a good approximation when the skin depth δ_{skin} is very small compared to the sheet thickness δ. Another possibility is the assumption that the field do not vary across the sheet [1]. These special elements (SE) are very easy to implement but only valid if the skin depth is large compared to the sheet thickness. For all other cases, higher order approximations are required, like ITCs using hyperbolic functions in thickness direction [2] (ITC-MB) or ITCs derived from prismatic elements where a polynomial of arbitrary order is considered in thickness direction (thin-sheet bases, TSB-<Order>). In taking the condition number of the system matrix into account, a new method [3] was derived by the authors that combines the advantages of [1] and [2]. In the new approach (MBF), the basis functions of the finite elements that are connected to the interface are modified in order to account for the variation in thickness direction.

Comparison

A 1D domain (μ_0, $\sigma = 0$) of 1m length including a sheet (μ_0, $\sigma = 10^7\,\text{Sm}^{-1}$, $\delta = 1\text{mm}$) is exited by a surface current $J_s = \pm 1000\text{A/m}$ at the two boundaries. The condition number of the resulting system matrix is compared for all the thin sheet approaches with respect to the ratio $\delta_{\text{skin}}/\delta$. All approaches are in some ranges advantageous, in some not, but it is obvious that the new method (MBF) combines the advantages of SE and ITC-MB: It is as accurate as the ITC-MB but also as good in the conditioning as SE.



Method	rel. error $\mathrm{Re}\{E_z(-\frac{\delta}{2})\}$
SE	1
MBF	exact
TSB-2 (–)	0.18
TSB-4 (--)	0.0019
TSB-6 (···)	0.000003
ITC-MB	exact

Figure 2: Comparison of the condition number and the relative field error at the sheet boundary.

Summary

Different thin-sheet approaches are compared with respect to the conditioning of the final system of equations. From the findings, a new method was developed that performs better according to this measure.

References

[1] Nakata, T. and Takahashi, N. and Fujiwara, K. and Shiraki, Y., “3-D magnetic field analysis using special elements”, IEEE Trans. Magn. 1990, vol. 26, no. 5, p. 2379–2381

[2] Mayergoyz, I.D. and Bedrosian, G., “On calculation of 3-D eddy currents in conducting and magnetic shells”, IEEE Trans. Magn. 1995, vol. 31, no. 3, p. 1319–1324

[3] Trommler, J. and Koch, S. and Weiland, T., “A finite-element approach in order to avoid ill-conditioning in thin-sheet problems in frequency domain - Application to magneto-quasistatics”, JCAM 2012, vol. 236, no. 18, p. 4671–4680

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